

# Model-based Co-simulation of Flexible Robotic Systems

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## EXTENDED ABSTRACT

### 1 Introduction

Complex machine systems are typically composed of subparts with various physical properties. When modelling and simulating such systems, a monolithic solution, which describes the motion of all components together, can be difficult to achieve. Co-simulation provides an alternative approach, where each subsystem is modelled and simulated separately with its micro time-step. The subsystems only exchange information at communication points, which are separated by time intervals referred to as the macro time-steps. The discontinuity and time delay in information exchange can cause error and instability. Different approaches have been studied to stabilize co-simulation based coupling. Some commonly used co-simulation methods are signal-based, which usually approximate the interface variables inside the macro time-step using extrapolations. Such signal-based approaches are prone to inaccuracy and instability when the macro time-step is set to a relatively large value. On the other hand, model-based approaches determine the coupling information based on system models, which can improve the performance and stability of co-simulation.

The concept of the reduced interface model (RIM) has been introduced to model-based co-simulation in recent years. In such an approach, reduced-order models are created at the coupling interface to represent the dynamics of each subsystem. The RIMs are used inside the macro time-step to mimic the dynamic performance of the corresponding subsystems. Each RIM is updated at the communication points and is simulated together with the other subsystem during the macro time-step. The RIM-based co-simulation has been applied to different systems with contact [1], showing more stable performance than signal-based methods. While previous studies in co-simulation mostly focus on rigid multibody systems, many realistic mechanical system models have to account for structural properties and deformation. Therefore, it is important to develop a systematic co-simulation approach for general systems with flexibility. In this study, the concept of RIM-based co-simulation is extended to flexible systems. A space robotic model with contact operations is employed to illustrate the approach. Moreover, by comparing the difference between the RIMs of rigid and flexible models, we found that co-simulation with flexible RIM can resolve some instability problems observed in the rigid body models.

### 2 RIM-based co-simulation of flexible mechanical systems

The dynamics of a general mechanical subsystem with flexibility can be written in the form of

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{c} = \mathbf{f} = \mathbf{f}_i + \mathbf{f}_c + \mathbf{f}_e + \mathbf{f}_d + \mathbf{f}_o \quad (1)$$

where  $\mathbf{M}$  is the  $n \times n$  mass matrix;  $\mathbf{v}$  is the  $n \times 1$  array of generalized velocities;  $\mathbf{c}$  is the Coriolis and centrifugal terms; the generalized force  $\mathbf{f}$  consists of the interface force  $\mathbf{f}_i$ , the constraint force  $\mathbf{f}_c$ , the elastic force  $\mathbf{f}_e$ , the damping force  $\mathbf{f}_d$ , and other applied forces represented by  $\mathbf{f}_o$ . If the deformation is small, which is common for the structural components in mechanical systems, floating frame of reference formulation may be applied [2]. In that approach, the motion of the system is separated to rigid body motion and deformation with an additive decomposition. In this case, the generalized velocity can be written as  $\mathbf{v} = [\mathbf{v}_r^T \ \mathbf{v}_e^T]^T$ , where  $\mathbf{v}_r$  is an  $n_r \times 1$  array representing the rigid body motion,  $\mathbf{v}_e$  is an  $n_e \times 1$  array describing the deformation, where  $n_r$  and  $n_e$  are the numbers of generalized velocities needed to describe rigid body motion, and flexible deformations, respectively. For a rigid body system model,  $n_e = 0$ , and the terms related to deformation vanish in Eq. (1).

When the system is subjected to constraints, the constrained velocities satisfy  $\mathbf{w}_c = \mathbf{A}_c \mathbf{v}$ , where  $\mathbf{A}_c$  is the constraint Jacobian matrix. For common bilateral constraints,  $\mathbf{w}_c = \mathbf{0}$ .

In co-simulation, when the coupling interface is properly defined, the interface velocities are related to  $\mathbf{v}$  as  $\mathbf{w}_i = \mathbf{A}_i \mathbf{v}$ , where  $\mathbf{A}_i$  is an  $n_i \times n$  interface Jacobian matrix. The dynamic equation describing the motion in the interface space [3] can then be written in the form of

$$\mathbf{M}_{eff} \dot{\mathbf{w}}_i + \mathbf{z}_i = \boldsymbol{\lambda}_i + \tilde{\boldsymbol{\lambda}} \quad (2)$$

where

$$\mathbf{M}_{eff} = (\mathbf{A}_i (\mathbf{I} - \mathbf{P}_c) \mathbf{M}^{-1} \mathbf{A}_i^T)^{-1} \quad (3)$$

$$\tilde{\boldsymbol{\lambda}} = \mathbf{M}_{eff}[\mathbf{A}_i(\mathbf{I} - \mathbf{P}_c)\mathbf{M}^{-1}(\mathbf{f}_e + \mathbf{f}_d + \mathbf{f}_o)] \quad (4)$$

$$\mathbf{z}_i = \mathbf{M}_{eff}[\mathbf{A}_i(\mathbf{I} - \mathbf{P}_c)\mathbf{M}^{-1}\mathbf{c} - \dot{\mathbf{A}}_i\mathbf{v} + \mathbf{A}_i\mathbf{M}^{-1}\mathbf{A}_c^T(\mathbf{A}_c\mathbf{M}^{-1}\mathbf{A}_c^T)^{-1}\dot{\mathbf{A}}_c\mathbf{v}] \quad (5)$$

In Eq. (3)-(5),  $\mathbf{P}_c = \mathbf{M}^{-1}\mathbf{A}_c^T(\mathbf{A}_c\mathbf{M}^{-1}\mathbf{A}_c^T)^{-1}\mathbf{A}_c$  is the projector matrix onto the subspace of constraint motion;  $\mathbf{M}_{eff}$  is the  $n_i \times n_i$  effective mass matrix;  $\boldsymbol{\lambda}$  is the interface force;  $\tilde{\boldsymbol{\lambda}}$  is the effective force;  $\mathbf{z}_i$  is the non-linear inertia terms.

In RIM-based coupling, at each communication point, instead of exchanging information directly, the dynamic information of each subsystem is used to update the corresponding RIM based on Eqn. (3)-(5). The dynamic equation of RIM in the form of Eqn. (2) is then integrated together with the full-order model of the other subsystem in the succeeding macro time-step.

### 3 Case study and analysis

A robotic manipulator model is employed here to represent the RIM-based co-simulation approach. The model is divided into the robotic arm subsystem and the payload subsystem with the interface defined at the centre of mass of the end-effector. The robotic arm has a symmetric architecture connected by seven revolute joints. In RIM-based co-simulation, a RIM is created at the communication points to represent each subsystem and is simulated together with the other side during the macro time-step. In this work, two robotic arm models were developed, one containing only rigid links, the other taking into account structural flexibility. Both models can be co-simulated with the payload subsystem through RIM-based coupling, leading to rigid and flexible RIMs respectively. A manoeuvre as Fig. 1a is designed for tests, where the payload first moves directly to the right-hand side, then rotates 90 degrees while reaching the surface of the wall, and finally stays in contact with the wall. To introduce a more challenging case for RIM-based coupling, the second joint at the shoulder of the robotic arm is locked during the manoeuvre to reduce one degree of freedom and to create near-singular configurations.

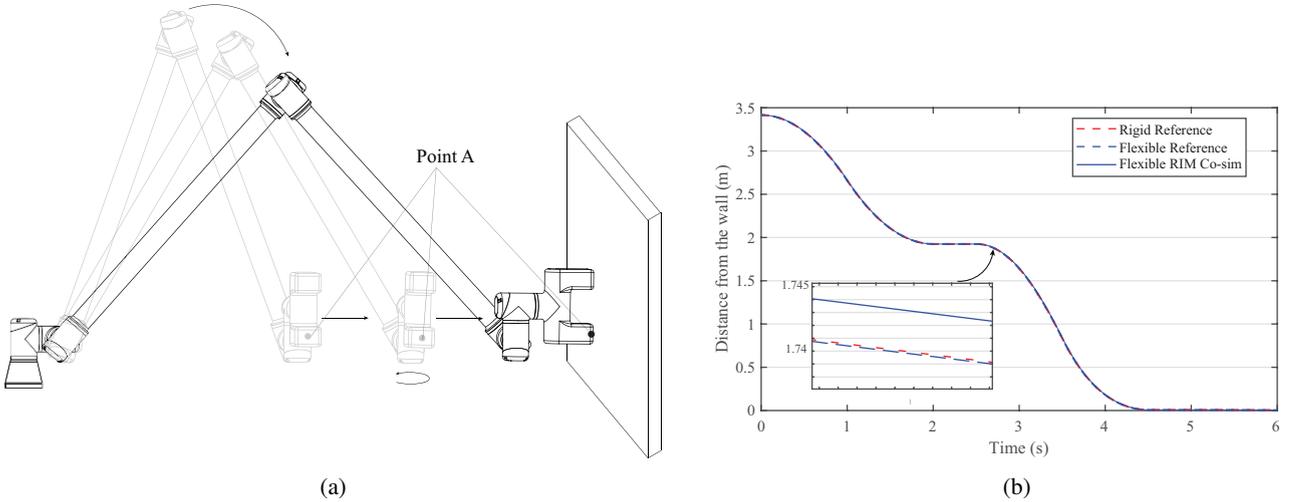


Figure 1: (a) Designed manoeuvre of the robotic arm (b) Distance between Point A and the wall

The distance between Point A and the wall in the simulation is compared, where monolithic solutions are used as references. The co-simulation employing the RIM based on the rigid body model fails during the simulation of this manoeuvre due to singularity in the model. On the contrary, the RIM based on the flexible multibody model resolves the problem and produces accurate results as shown in Fig. 1b. This demonstrates that including structural flexibility in the RIM can solve some potential singularity and instability problems of co-simulation.

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